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HEAT AND MASS TRANSFER WITH TRANSPIRATION COOLING

A. V. LUIKOV

Heat and Mass Transfer Institute of the B.S.S.R. Academy of Sciences, Minsk, B.S.S.R., U.S.S.R.

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Аннотация—Дан анализ влияния поперечного потока массы на теплообмен при ламинарном обтекании влажной капиллярно-пористой пластины. Установлены закономерности тепло-и массообмена при испарении жидкости из капиллярно-пористых тел. Показано, что в случае углубления поверхности испарения коэффициенты теплообмена больше по сравнению с коэффициентами теплообмена при испарении на поверхности тел.

Наблюдаемые в некоторых работах по испарительному пористому охлаждению уменьшения коэффициента теплообмена с увеличением интенсивности испарения объясняются методикой расчёта.

NOMENCLATURE

a,	thermal diffusivity coefficient ($a = kc_p\rho$);
c_p,	specific heat, at constant pressure;
h,	heat-transfer coefficient;
D,	vapour diffusion coefficient in moist air;
j_1,	evaporation intensity or mass flow density;
k,	heat conduction coefficient;
l,	characteristic body dimension;
p,	total pressure of moist air ($p = p_1 + p_2$ = const.);
q,	heat flux;
r,	latent heat;
t,	temperature °C;
T,	absolute temperature °K;
w,	air velocity;
δ,	boundary-layer thickness;
η,	dynamic viscosity coefficient;
ν,	kinematic viscosity coefficient ($\nu = \eta/\rho$);
ρ,	density;
ρ_{10},	relative vapour concentration ($\rho_{10} = \rho_1/\rho$);
τ,	time;
ξ,	distance of evaporation surface from body surface;
ϕ,	relative air humidity.

Dimensionless numbers

Re_x,	local Reynolds number [$Re_x = (w_x x/\nu)$];
Pe_x,	local Peclet number ($Pe_x = (w_x x/a)$];
$\overline{Pe}_x$,	local Peclet number relative to mean integral velocity $\bar{w}_x$ in a boundary layer ($\overline{Pe}_x = \bar{w}_x x/a$);
Nu_x,	local Nusselt number [$Nu_x = (h_x x/k)$];
Sc,	Schmidt number [$Sc = (\nu/D)$];
Pr,	Prandtl number [$Pr = (\nu/a)$];
Gu,	Gukhman number ($Gu = T_a - T_b/T_a$).

Subscripts and Superscripts

a,	surrounding medium (moist air);
b,	adiabatic saturation state;
e,	heat transfer with evaporation;
s,	surface;
x,	local value depending on x co-ordinate;
1,	vapour;
2,	dry air;
',	solid body;
o,	heat transfer without evaporation.

INTRODUCTION

HEAT and mass transfer with liquid evaporation from capillary-porous bodies is not only of theoretical interest but of great practical importance in engineering.

A heat- and mass-transfer process between a body surface and the surrounding medium is inseparably linked with that inside a capillary-porous body.

Up to the present day opinion is divided on the subject of the physical mechanism of heat and mass transfer and also on the qualitative effect of mass transfer upon heat transfer.

On the basis of the theory of gas injection into a boundary layer through a porous wall some investigators consider that with liquid evaporation the heat-transfer coefficient decreases with an increase in evaporation intensity and with vapour condensation it increases with condensation.†

In the first case a transverse mass flow is directed in an opposing sense to the heat flow and in the second case these flows have the same direction.

The boundary-layer thickness then increases with evaporation (boundary layer "swells") and this leads to a decrease in the heat-transfer coefficient. With vapour condensation a different process takes place which leads to an increase in the heat-transfer coefficient with condensation rate.

However, experiments on liquid evaporation from an open surface, made by Nesterenko [1], Sergeyev [23] and other investigators, showed that heat-transfer coefficients with evaporation are greater than h_e, as compared with those without evaporation under other equal hydrodynamic conditions and with the same temperature differences. This difference $(h_e - h_0)$ increases with a decrease in relative air humidity.

When drying moist materials, the heat-transfer coefficients are greater than those of a dry body [3–6].

In the process of transpiration cooling the results differ. In [7] the heat-transfer coefficients increase with evaporation rate whilst in another case, on the contrary, they decrease [8].

In the present paper an attempt is made to explain these contradictory results on the basis of the theory of submerging evaporation surface in capillary-porous bodies.

POROUS COOLING WITH GAS INJECTION INTO BOUNDARY LAYER

Porous cooling by gas injection into a boundary layer was calculated in detail by Eckert [9].

This calculation is based on the solution of a system of differential heat- and mass-transfer equations for a flat porous plate in a laminar boundary layer of a binary gas mixture.

The system of differential equations is as follows:

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0, \tag{1}$$

$$\rho w_x \frac{\partial w_x}{\partial x} + \rho w_y \frac{\partial w_x}{\partial y} = \frac{\partial}{\partial y}\left(\eta \frac{\partial w_x}{\partial y}\right), \tag{2}$$

$$c_p \rho w_x \frac{\partial t}{\partial x} + c_p \rho w_y \frac{\partial t}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial t}{\partial y}\right) + \rho D(c_{p1} - c_{p2}) \frac{\partial \rho_{10}}{\partial y} \frac{\partial t}{\partial y}, \tag{3}$$

$$\rho w_x \frac{\partial \rho_{10}}{\partial x} + \rho w_y \frac{\partial \rho_{10}}{\partial y} = \frac{\partial}{\partial y}\left(D\rho \frac{\partial \rho_{10}}{\partial y}\right). \tag{4}$$

In this case thermal diffusion (the Soret effect) and diffusion heat conduction (the Dufour effect) are neglected as they are small.

Boundary conditions are as follows:

at $y = 0$, $w_x = 0$, $w_y = w_s$; $t = t_s$, $\rho_{10} = \rho_{10s}$ (5)

at $y = \infty$, $w_x = w_a$, $t = t_a$, $\rho_{10} = \rho_{10a}$. (6)

Moreover, it was assumed that the linear transverse velocity changed along a surface (in x-direction) in inverse proportionality to $\sqrt{x}$.

Calculation results for a mass flow of injected gas which is directed from a body surface (analog of evaporation process) are given in Fig. 1.

From Fig. 1 it is evident that heat- and mass-transfer coefficients decrease with an increase in parameter Z.

$$Z = \frac{w_s}{w_a}\sqrt{(Re_x)} = \frac{j_1}{\rho w_a}\sqrt{(Re_x)}, \tag{7}$$

where $j_1 = \rho w_s$ is the mass-transfer rate in the direction of a normal to the wall surface (in y-direction). Hence, the heat-transfer coefficient decreases with an increase in the

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† It is assumed that heat necessary for evaporation is transferred to a body from the heated air by convection.

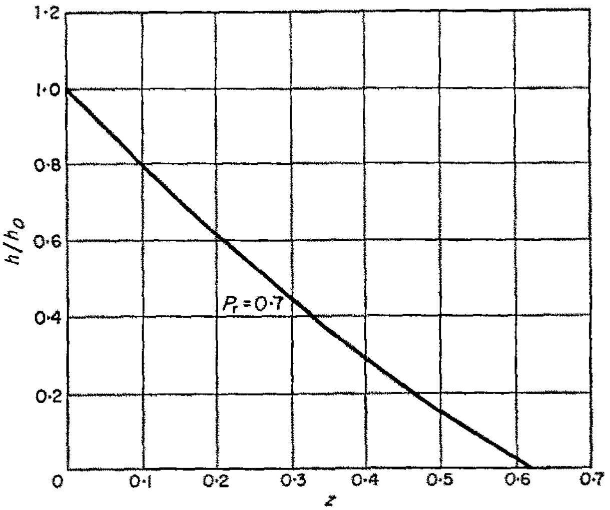

FIG. 1. Relation between h/h_0 and parameter z with inert gas injection ($Pr = 0{\cdot}7$) into a boundary layer according to Eckert's data.

transverse mass-flow rate. With $Z = 0{\cdot}05$ the ratio h/h_0 is equal to 0·9 (see Fig. 1). Consequently, with $Z = 0{\cdot}05$ the decrease in the heat-transfer coefficient, h, compared with that for a dry body, h_0, is 10 per cent. A decrease by a factor of two ($h/h_0 = 0{\cdot}5$) corresponds to $Z = 0{\cdot}3$.

If the liquid-evaporation process is considered to be the analog of a process of gas injection into a boundary layer, then under adiabatic evaporation conditions the parameter Z may be presented in another form. With convective heat transfer, evaporation intensity j_1 is equal to:

$$j_1 = \frac{h_x}{r}(t_a - t_b) = \frac{k}{xr} Nu_x(t_a - t_b). \tag{8}$$

When a plate is in a laminar flow of moist air, the local Nusselt number, Nu_x, is equal to:

$$Nu_x = A_x \sqrt{Re_x}$$

where A_x is the constant.

Then, the parameter Z will be:

$$Z = \frac{A_x}{Pr}\frac{c_p \Delta t}{r} = \frac{A_x}{Pr}\frac{c_p T_a}{r} Gu. \tag{9}$$

Consequently, the parameter Z is independent of the air velocity but it depends on a psychrometric difference $\Delta t (\Delta t = t_a - t_b)$ or on the Gukhman number.

Let us do some approximate calculations for conditions: $t_b = 30$°C, $r = 579$ kcal/kg, $A_x = 0{\cdot}33$, $Pr = 0{\cdot}7$. Then for $Z = 0{\cdot}05$ the temperature drop will be equal to $\Delta t = 260$°C. Consequently, only at the temperature difference of $\Delta t \geqslant 260$°C will a decrease in the heat-transfer coefficient be of order 10 per cent.

Similar results were reported in [8] when a porous plate was located in a flow of heated air. In these experiments the evaporation rate did not exceed 22·7 kg/m^{2}h. The Reynolds number ranged from 10^4 to $2{\cdot}10^6$ and air temperature, from 18 to 140°C. Evaporation occurred under adiabatic conditions. The temperature difference did not exceed 80°C. Hence, the transverse mass flow could not influence the heat-transfer coefficient greatly.

However, the authors established the following relation for the mean Nusselt number:

$$Nu = 0{\cdot}00455\, Re^{0{\cdot}8} \left(\frac{c_p \Delta t}{r}\right)^{-0{\cdot}4}. \tag{10}$$

Thus, the heat-transfer coefficient decreases by a factor of more than three.

Although the analysis of [8] is of a tentative character, since water evaporation from the porous plate occurred in a turbulent flow, the decrease in the heat-transfer coefficient, however, with increase in the temperature difference, Δt, proceeded due to other reasons which will be considered below.

TRANSPIRATION COOLING

The Krischer method [10] is used to analyse a complex heat-transfer problem with transpiration cooling. The essence of this method lies in the following.

If the influence of transverse mass flow is neglected, then the differential heat-transfer equation for a flat plate in a laminar boundary layer and at constant transfer coefficients may be written thus (see Appendix):

$$w_x \frac{\partial t(x,y)}{\partial x} = a \frac{\partial^2 t(x,y)}{\partial y^2}. \qquad (11)$$

Boundary conditions are taken in the form:

$$\text{at } y = 0,\ t(x,0) = t_s = \text{const.} \qquad (12)$$

$$\text{at } x = 0,\ t(0,y) = t_a = \text{const.} \qquad (13)$$

$$\text{at } y \to \infty,\ t(x,\infty) = t_a = \text{const.} \qquad (14)$$

The flow velocity, w_x, is a function of the coordinates; it is determined from equations (1–2). Following the Krischer method, w_x is assumed to be constant and equal to the mean flow velocity in a boundary layer ($w_x = \bar{w}_x = \text{const.}$).

In actual processes the velocity is constant only when a body is in a flow without friction, i.e. at a very small coefficient of internal friction.

In case of viscous liquid the assumption ($w_x = \bar{w}_x = \text{const.}$) is a certain method for solving a heat-transfer problem in a boundary layer.

When an infinitely long plate is placed in a flow ($l = x \to \infty$), the solution of differential equation (11) at boundary conditions (12–14) has the form:

$$\frac{t(x,y) - t_s}{t_a - t_s} = \operatorname{erf}\left(\frac{y\sqrt{(\bar{w}_x)}}{2\sqrt{(ax)}}\right). \qquad (15)$$

The local Nusselt number, Nu_x, is equal to

$$Nu_x = \frac{h_x x}{k} = \frac{x}{(t_a - t_s)} \frac{\partial t(x,0)}{\partial y}. \qquad (16)$$

Differentiating solution (15) with respect to y and assuming $y = 0$, we obtain:

$$Nu_x = \frac{1}{\sqrt{\pi}} \sqrt{\left(\frac{\bar{w}_x x}{a}\right)} = \frac{1}{\sqrt{\pi}} \sqrt{(\overline{Pe}_x)}. \qquad (17)$$

The mean Nusselt number on a surface is

$$Nu = \frac{1}{l} \int_0^l \frac{l}{x} Nu_x \, dx = \frac{2}{\sqrt{\pi}} \sqrt{(\overline{Pe})}, \qquad (18)$$

where $\bar{P}e = (\bar{w}_x l/a)$ is the mean Peclet number based on the mean velocity.

To compare the result obtained with the known formulae for Nu for a plate in a laminar flow, it is necessary to determine $\bar{w}_x$.

If the velocity profile $w_x(y)$ is assumed to be a cubic parabola, then the mean velocity w_x is equal to

$$\bar{w}_x = w_a \frac{1}{\delta} \int_0^\delta w_x(y)\, dy = \frac{5}{8} w_a. \qquad (19)$$

Then for humid air ($Pr = 0{\cdot}7$) we have

$$Nu = \frac{2}{\sqrt{\pi}} \cdot 0{\cdot}625 \sqrt{(Pe)} = 0{\cdot}74 \sqrt{(Re)}. \qquad (20)$$

This result differs from the known formula for a flat plate only by 20 per cent, $Nu = 0{\cdot}60 \sqrt{(Re)}$.

Thus, the assumption of constant velocity, w_x, when solving equation (11) yields satisfactory results.

Our problem on transpiration cooling may be presented as follows (see Fig. 2).

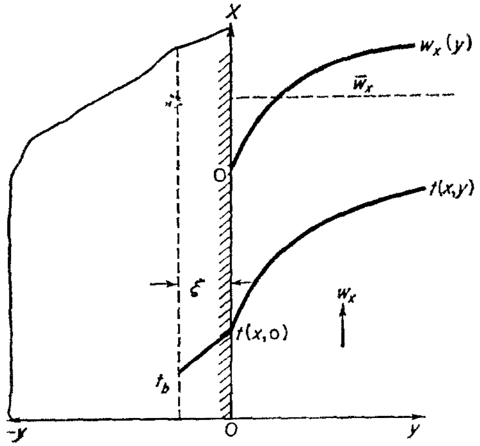

FIG. 2. Diagram of heat-transfer calculation with deepening of evaporation surface.

The differential heat-transfer equation for a boundary layer remains just the same [see equation (11)].

Boundary conditions are:

$$\text{at } x = 0: t(0,y) = t_a, \text{ at } y = -\xi,\ t(x_1 - \xi) = = t_b = \text{const.} \quad (21)$$

$$\text{at } y \to \infty\ t(x, \infty) = t_a \quad (22)$$

$$\text{at} \quad y = -\xi, \quad -k\frac{\partial t(0,x)}{\partial y} = -k'\frac{\partial t(0,x)}{\partial y}. \quad (23)$$

In this case the temperature on the evaporation surface ($y = -\xi$) is assumed to be equal to the wet-bulb temperature. Owing to the small thickness of the boundary layer on the body ξ the temperature distribution is assumed linear. Then boundary condition (23) may be written thus:

$$-\frac{\partial t(0,x)}{\partial y} + H[t(0,x) - t_b] = 0 \quad (23a)$$

where $H = (k'/k\xi)$ is some quantity analogous to a relative heat-transfer coefficient.

The solution of differential equation (11) at boundary conditions (21–23) has the form

$$\frac{t(x,y) - t_b}{t_a - t_b} = \operatorname{erf}\left[\frac{y\sqrt{\bar{w}_x}}{2\sqrt{(ax)}}\right] + \exp\left(Hy + H^2\frac{ax}{\bar{w}_x}\right)\operatorname{erfc}\left[\frac{y\sqrt{\bar{w}_x}}{2\sqrt{(ax)}} + \sqrt{\frac{(ax)}{\bar{w}_x}}\right]. \quad (24)$$

Solution (15) is obtained from that of (24) as a particular case.

If evaporation proceeds on a body surface ($\xi = 0$), then $H \to \infty$ and the second term of the right-hand side of solution (24) will be equal to zero.

Temperature on a body surface, $t_s[t_s = t(0,x)]$ will not be constant and changes along the x-axis.

$$\frac{t(x,0) - t_b}{t_a - t_b} = \exp\left(H^2\frac{ax}{\bar{w}_x}\right)\operatorname{erfc}\left(\sqrt{\frac{(ax)}{\bar{w}_x}}\right). \quad (25)$$

Near the edge of a plate ($x = 0$) temperature of a body surface, t_s, is equal to that of air ($t_s - t_a$) and at a considerable distance from the edge ($x \to \infty$) the temperature of a body surface is equal to the wet-bulb temperature ($t_s - t_b$). Consequently, the temperature difference $\Delta t(\Delta t = t_a - t_s)$ changes from zero near the plate edge to a constant value ($t_a - t_b$).

This is a very important fact which defines heat and mass transfer when submerging the evaporation surface deep into a body.

If evaporation occurs on a body surface, then under adiabatic conditions temperature of the body surface is constant and equal to the wet-bulb temperature.

From the theory of heat transfer it is known that if temperature difference increases along the flow, then the heat-transfer coefficient is greater than that with constant temperature difference.

Consequently, when deepening an evaporation surface, the heat-transfer coefficient is greater as compared with that with evaporation on a surface.

If to a first approximation it is assumed that the heat-transfer coefficient with evaporation on a body surface is equal to that of a dry body, then with transpiration cooling, when deepening the evaporation surface, the heat-transfer coefficient will be greater in comparison with that of a dry body. Consider it in detail.

The local Nusselt number, Nu_x, is equal

$$Nu_x = \frac{x}{[t_a - t(x,0)]}\frac{\partial t(x,0)}{\partial y} = \sqrt{(Pe_x)}K \exp K^2 \operatorname{erfc} K(1 - \exp K^2 \operatorname{erfc} K]^{-1} \quad (26)$$

where K is a dimensionless parameter

$$K = \frac{Hx}{\sqrt{(Pe_x)}} = \frac{k^1}{k\xi}\left(\frac{ax}{\bar{w}_x}\right)^{0\cdot5}. \quad (27)$$

Designate

$$f(K) = \sqrt{(\pi)}\, K \exp K^2 \operatorname{erfc} K, \quad (28)$$

then we have

$$N = \frac{\sqrt{(\pi)}\, Nu_x}{\sqrt{(Pe_x)}} = \frac{f(K)}{1 - 1/[K\sqrt{(\pi)}]f(K)}. \quad (29)$$

(17) is obtained from (29) as a specific case. If evaporation occurs on a body surface ($\xi = 0$, $K = \infty$), then $N = 1$, since $f(K) = 1$.

Thus, the dimensionless quantity N characterizes a relative increase in Nu_x with liquid evaporation from a capillary-porous body at some depth ξ, as compared with evaporation on a body surface. In Fig. 3, N is plotted versus K, from which it is seen that N decreases with an increase in K (at $K \to \infty$ $N \to 1$).

Over a range of values of K from 0·1 to 5·0 ($0{\cdot}1 < K < 5{\cdot}0$) the relation $N = F(K)$ may be approximated by the following formula

$$N = 1{\cdot}31\, K^{-0{\cdot}12}. \tag{30}$$

From the theory of liquid transfer in capillary-porous bodies it is known that for the first approximation ξ is proportional to a relative psychrometric difference $(T_a - T_b)/T_a$.

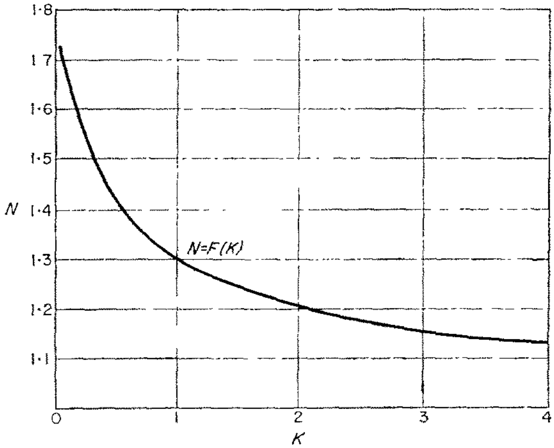

FIG. 3. Relation between coefficient N and parameter K.

Then the dimensionless parameter K will be inversely proportional to $(T_a - T_b)/T_a$ or to the Gukhman number ($K \sim 1/Gu$). Hence, it follows that N will be directly proportional to $Gu^{0{\cdot}12}$ ($N \sim Gu^{0{\cdot}12}$). This obtains in some works on heat transfer with drying [2–5].

It is of interest to determine ξ roughly. For porous ceramics used in [2] $k = 0{\cdot}2$ kcal/mh degC and $Re = 6{\cdot}10^4$, $Pr = 0{\cdot}7$. Then, for $x = 20$ mm $K = 1{\cdot}5$ and $\xi = 0{\cdot}5$ mm. In this case according to the plot in Fig. 3 the coefficient $N = 1{\cdot}25$, which corresponds to an increase in the heat-transfer coefficient, h_e, by 25 per cent as compared with that for a dry body ($h_e/h_0 = 1{\cdot}25$) if heat transfer with evaporation on a body surface is considered to be identical with that of a dry body.

For $K = 0{\cdot}25$, $\xi = 3$ mm and $N = 1{\cdot}55$. Hence, the heat-transfer coefficient, h_e, is approximately greater by 55 per cent than h_0. It is quite natural that at small values of ξ the temperature of a body surface cannot be measured in practice. Thermocouples embedded on a "surface" of a body in fact show the wet-bulb temperature t_b. The heat transfer coefficient is, therefore, calculated as the relation of a heat flow, q, to the psychrometric difference $(t_a - t_b)$.

$$h_{xb} = \frac{q}{t_a - t_b} \tag{31}$$

In this case the local Nusselt number, Nu_{xb} will be

$$Nu_{xb} = \frac{h_{xb}x}{k} = \frac{x}{(t_a - t_b)} \frac{\partial t(x,0)}{\partial y}. \tag{32}$$

After simple transformations we have:

$$N_b = \frac{\sqrt{(\pi)}\, Nu_{xb}}{\sqrt{(\overline{Pe}_{xb})}} = f(K) \tag{33}$$

where N_b is a coefficient showing a relative change in Nu_{xb} and, consequently, h_{xb} due to the deepening of an evaporation surface.

From the plot $N_b = f(K)$ depicted in Fig. 4 one can see that the coefficient K increases with N_b.

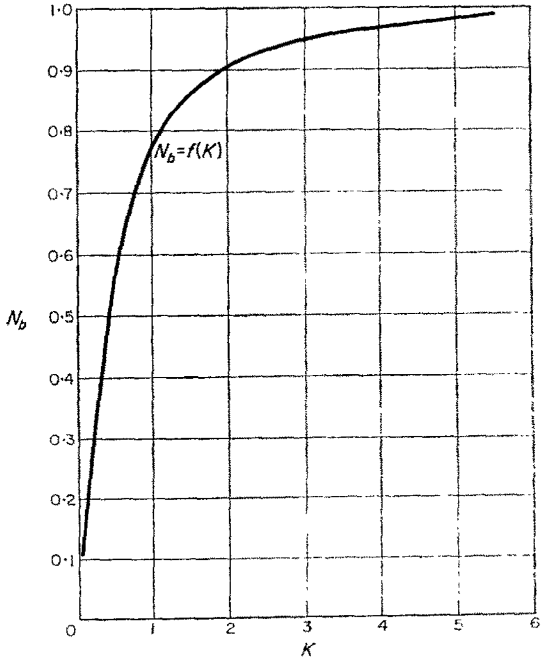

FIG. 4. Relation between coefficient N_b and parameter K.

Since K is inversely proportional to the Gukhman number, Nu_{xb} will decrease with an increase in Gu and this is reported in [8].

Over a small range of change in K the relation $N_b = f(K)$ may be represented by the formula

$$N_b = BK^m \tag{34}$$

where B and m are constants ($1 > m > 0$).

Over the range ($0{\cdot}3 < K < 1{\cdot}5$) $B = 0{\cdot}73$ and $m = 0{\cdot}46$. In the range ($1{\cdot}5 < K < 5$) $B = 0{\cdot}80$ and $m = 0{\cdot}15$ (see Fig. 4). Hence, in the range ($0{\cdot}3 < K < 1{\cdot}5$) we have

$$N_b = 0{\cdot}73\, K^{0{\cdot}46} = B_1\, Gu^{-0{\cdot}46} \tag{35}$$

where B_1 is the constant.

This formula by its structure is close to equation (10). It may be noted that the authors of [8] treated their experimental data using the Gukhman number. After this they obtained the following dependence:

$$\frac{Nu}{Re^{0{\cdot}8}} = 0{\cdot}00695\, Gu^{-0{\cdot}5}. \tag{36}$$

It should be noted that the comparison of the above formulae is of a conditional character, since ξ depends on the capillary-porous structure of a body, its physical-chemical properties and in general case is a function of the parametric number (T_a/T_b). However, equations (33), (34), (10) and (36) cogently show that in [8] evaporation proceeded at some depth from a body surface. Decrease in the heat-transfer coefficient due to increase in the evaporation rate, observed in experiments, is explained not by the effect of a transverse mass flow on boundary-layer thickness but by the method of calculation of the heat-transfer coefficient according to (31).

Data obtained in [11] may be cited as a second example. With drying of gypsum plates (200 × 50 × 25 mm) at air temperature $t_a = 33$°C, relative humidity $\phi = 20$ per cent and air velocity from 1 m/s to 5 m/s ($0{\cdot}9 \times 10^4 < Re < 3{\cdot}5 \times 10^4$) the following formula† may be obtained

$$Nu = 7{\cdot}25\, Re^{0{\cdot}4}. \tag{37}$$

The value of the exponent of Re may be explained by the effect of the parameter K. Over the range ($1{\cdot}5 < K < 5{\cdot}0$) the local number, Nu_{xb} is equal to

$$Nu_{xb} = \frac{1}{\sqrt{(\pi)}}\, Pr^{0{\cdot}43} \left(\frac{H}{\xi}\right)^{0{\cdot}15} Re_x^{0{\cdot}43}. \tag{38}$$

The exponent of Re_x ($n = 0{\cdot}43$) is close to that of Re in (37).

EXPERIMENTS ON TRANSPIRATION COOLING

At the Heat and Mass Transfer Institute of the B.S.S.R. Academy of Sciences, Shulman carried out experiments on transpiration cooling [12–13]. A ceramic hollow cylinder (250 mm in height and 22 mm in internal radius) in the shape of a tumbler is taken as a sample.

A porous ceramic possesses a monocapillary structure (capillary radius, corresponding to a maximum of a distribution curve for pores, is equal to 0·9 μ), its total porosity is 70 per cent. Experiments were calculated at air temperatures; 70°C, 100°C and 130°C. The air velocity in the wind tunnel was from 6·0 to 150 m/s. Reynolds numbers ranged between $2{\cdot}10^4$ and $7{\cdot}10^4$.

Two grooves (2·5 mm in depth and 2 mm in width) were made on the internal wall of the tumbler. The distance between their axes was equal to 14·5 mm. A cardboard moisture-proof small boat was inserted into these grooves and fixed by glue. After heating and drying, the sealing of the boat and a great cavity of an experimental body was checked. Thermocouples were imbedded along the external contour and along the generating line of the porous cylinder to measure body temperature.

The lower semi-spherical part of the tumbler was soaked with water-proof varnish. Therefore, water evaporation proceeded only from the lateral cylinder side. To approach experimental conditions to a plane problem, the porous cylinder had a lengthening device made of cardboard. Water was fed into the boat and glass cavity from measuring burettes with scale division equal to 0·05 mm. The experimental body (porous cylinder) was placed in a wind tunnel, described in [5], normal to the air flow. The cylinder was filled with heated water. A specific heat flow for the given portion of the cylinder surface ($q = rj_1$) was determined by

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† Value, equal to $\sqrt{s}$, where s is the evaporation surface ($l\sqrt{s}$) was taken as the characteristic dimension l.

water rate in the boat. The boat axis is established at a given distance from the front critical point by turning a porous cylinder at a certain angle according to the limb. Measurements were made over the range of the angle θ between 0° and 180° through each 15° starting from the front critical point. Evaporation occurred under adiabatic conditions and the thermocouples, imbedded near the surface of the porous cylinder, showed temperatures equal to that of the wet-bulb ($T_s = T_b$). The heat-transfer coefficient was calculated from (36), i.e. the coefficient of heat transfer from air to an evaporating surface through the air boundary layer and a body layer was determined.

Experimental values of the parameter Z depending on the angle θ are shown in Fig. 5. From Fig. 5 one can see that it does not exceed 0·04 ($Z > 0{\cdot}04$). Hence the influence of a transverse mass flow on heat transfer is negligible.

For all the temperatures (70°, 100° and 130°C) the relation between the local numbers Nu_{xb} and Re_x are close to a parabolic one, i.e. Nu_{xb} is directly proportional to $\sqrt{Re_x}$. The experimental data, treated as a plot of $Nu_{xb}/\sqrt{Re_x}$ versus Gu, showed that $Nu_{xb}/\sqrt{Re_x}$ is inversely proportional to $Gu^{0{\cdot}4}$.

Fig. 6 gives the relation between $Nu_{xb}/\sqrt{Re_x}\,.\,Gu^{-0{\cdot}4}$ and the angle θ.

Designate the dimensionless co-ordinate x by x^* ($x^* = x/d$) where x is the arc length of the external cylinder contour with the central angle θ, calculated from the front critical point ($x = \frac{1}{2}d\theta$), d is the cylinder diameter.

Then, experimental data depicted in Fig. 6 are described by the following empirical formula:

$$Nu_{xb} = 2{\cdot}0\,[\exp(-2{\cdot}46\,x^*) + 0{\cdot}2\exp(-55\,x^*)]\sqrt{(Re_x)}\,Gu^{-0{\cdot}4}. \qquad (39)$$

From formula (39) it follows that the coefficient h_{xb} decreases with an increase in the Gukhman number. The heat-transfer coefficient, h_x, increases

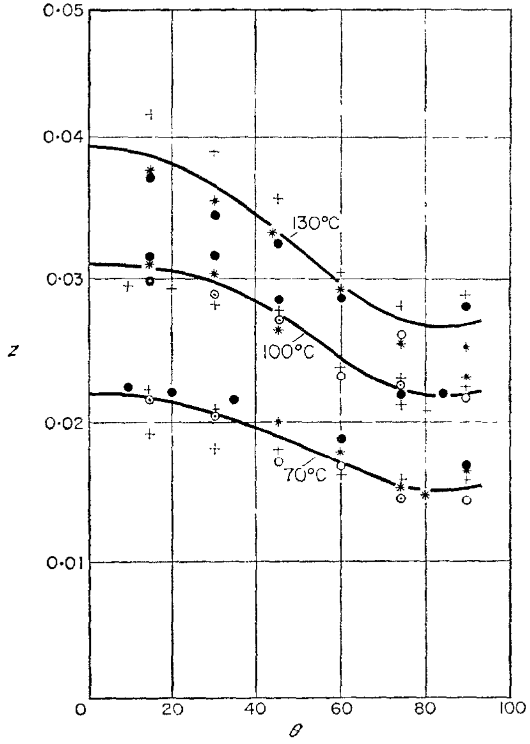

FIG. 5. Relation of parameter z and angle θ for various temperatures of air flow.

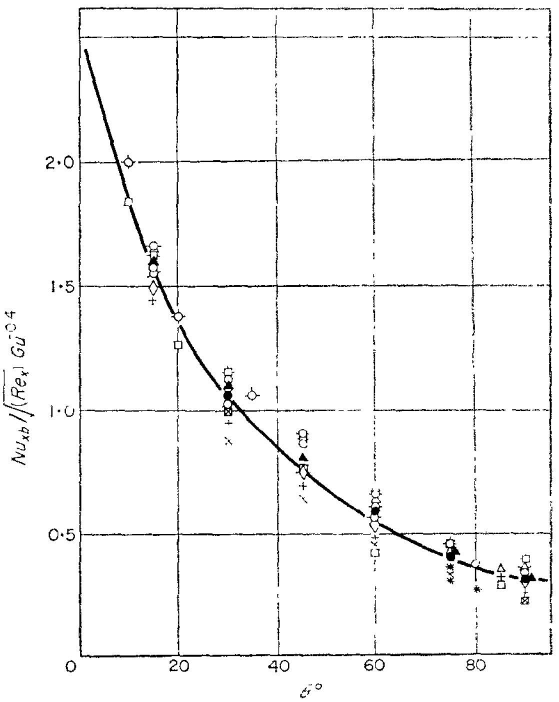

FIG. 6. Relation between local Nusselt number and and angle θ.

with Gu. There is a simple relation between the coefficients h_x and h_{xb}:

$$\frac{1}{h_{xb}} = \frac{1}{h_x} + \frac{\xi}{k'}. \tag{40}$$

To the first approximation the depth of position of the evaporation surface ξ is proportional to Gu but the coefficient h_{xb} decreases with an increase in Gu.

In Fig. 7 the average Nusselt numbers, Nu_b, over the cylinder surface for three temperatures are plotted versus Re. Values of Nu_b for dry heat transfer [14] are also given in Fig. 7. From Fig. 7 it follows that the Nusselt numbers for dry heat transfer coincide with those for moist heat transfer at an air temperature of 100°C ($Nu_0 = Nu_b$). At air temperature 130°C the values are smaller and at temperature 70°C are greater, as compared with data in [14]. More intense heat transfer between a moist porous body and air flow is explained by other reasons. Volumetric evaporation connected with the dynamic character of sorption and desorption processes is one of the reasons for increased heat and mass transfer.

The essence of this hypothesis lies in the fact that fine liquid drops enter a boundary layer. A process of nucleate vapour condensation and interaction between an air flow and body surface is the basic reason of droplet separation from a body surface. According to the dynamic adsorption theory [15] the evaporation process is a dynamic process of desorption and sorption. Liquid molecules do not only leave the evaporation surface but continuously return back to it, forming condensation nuclei. The evaporation rate is proportional to the difference between molecular flows leaving the surface and returning back to it. Fedyakin's investigations [16] showed that condensation does not occur uniformly along a body surface but on some portions (nuclei). In addition, incomplete wetting of a body surface proceeds over a condensation portion. On this portion drops are formed which are carried away by air flow into a boundary layer as they are less firmly connected with the body. The nuclear condensation and evaporation process breaks the structure of the boundary layer adjacent to a wall and this also leads to intensification in heat and mass transfer.

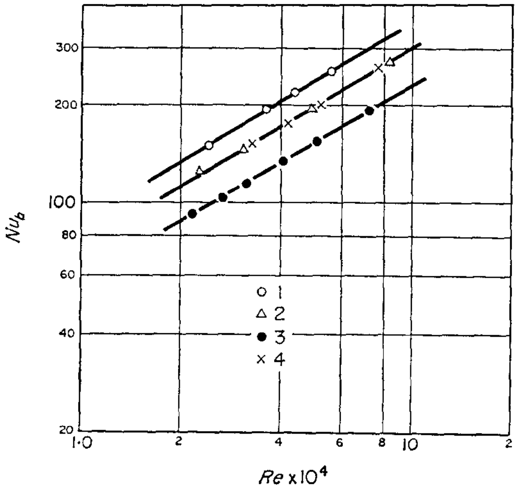

FIG. 7. Relation between mean Nusselt number Nu_b and Re at temperature of air flow: 1. 70°C, 2. 100°C, 3. 130°C, 4. dry heat transfer according to Kruzhilin and Schwab's data [14].

Evaporation of drops in a boundary layer is called volumetric evaporation; it is a vapour source and a negative heat source in the equations of heat and mass transfer of a boundary layer.

With volumetric evaporation the right-hand side of equation (4) should have the additional term, I_1, where I_1 is the volumetric vapour source (kg/m^{3}h). The right-hand side of equation (3) should embody the new term rI_1 which is a heat sink (kcal/m^{3}h). In [17] it is reported that volumetric evaporation in a boundary layer is characterized by the Gukhman number. Thus, the Gukhman number characterizes not only the effect of deepening an evaporation surface inside the body upon heat and mass transfer in a boundary layer but also upon a process of volumetric evaporation of drops in a boundary layer.

From Fig. 7 it follows that at an air temperature 70°C heat-transfer intensification due to volumetric evaporation will be considerably greater, as compared with the effect of the deepening of the evaporation surface (value ξ is small). Values of Nu_b are therefore greater than the corresponding values of Nu_0. At a temperature of 100°C the influence of these two effects is approximately the same. Therefore, the numbers Nu_0 and Nu_b coincide. At a temperature of 130°C the effect of the deepening of the evaporation surface suppresses that of ordinary evaporation, which leads to the relation $Nu_b < Nu_0$.

The basic conclusion of our work is as follows: heat and mass transfer of capillary-porous bodies within a surrounding medium is the interconnected heat- and mass-transfer process in a boundary layer of air and in that of a capillary-porous body. This process may be studied with advantage provided that it is considered as a single whole, as the whole complex of phenomena in their interaction.

Neither conclusions, based on analogy, nor quantitative laws based on the principle of additivity of relations, corresponding to practical phenomena, are possible. Correct quantitative relations should correspond to the actual process.

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APPENDIX

Equation (11) is a particular case of equation (3) when the transverse velocity is zero ($w_y = 0$). Equation (3) may be solved if we assume that $w_x = \bar{w}_x = \text{const.}$ and $w_y = j_s/\rho = \text{const.}$ Heat transfer due to diffusion is neglected. The solution of equation (3) with boundary conditions (12–14) is of the form

$$\frac{t(x, y) - t_s}{t_a - t_s} = 1 - \tfrac{1}{2}\left\{\operatorname{erfc}\left[\frac{\sqrt{(\bar{w}_x)}}{2\sqrt{(ax)}}y - \frac{w_y}{2a}\sqrt{\left(\frac{ax}{w_x}\right)}\right] + \exp\left(\frac{w_y}{a}y\right) \times \operatorname{erfc}\left[\frac{\sqrt{(\bar{w}_x y)}}{2\sqrt{(ax)}} + \frac{w_y\sqrt{(ax)}}{2a\sqrt{(w_x)}}\right]\right\}. \qquad \text{(I)}$$

From the above solution the local Nusselt number is obtained

$$Nu_x = \frac{1}{\sqrt{\pi}}\sqrt{(\overline{Pe}_x)}\exp\left(-\frac{Pe_x^{*2}}{4\overline{Pe}_x}\right) - \tfrac{1}{2}Pe_x^*\operatorname{erfc}\left[\tfrac{1}{2}Pe_x^*/\sqrt{(\overline{Pe}_x)}\right], \qquad \text{(II)}$$

where

$$Pe_x^* = \frac{w_y x}{a} = \frac{j_s x}{\rho a}$$

is the local Peclet number for transverse transfer. If the effect of convective heat transfer along y is neglected ($Pe_x^* = 0$), from equation (II) formula (17) is obtained. If the evaporation rate does not exceed 20 kg/m²h ($j_s < 20$), then for a wet plate in a laminar air flow ($Pr = 0{\cdot}7$) $Pe_x^* < 25$ when $Re \leqslant 8{\cdot}10^4$. Hence the second term of formula (II) is less than 5 per cent in relation to the first term, and the value of $\exp(-Pe_x^{*2}/4\overline{Pe}_x)$ is practically unity.

The solution of equation (3) with boundary conditions (21–23) is of the form

$$\left.\begin{aligned}\frac{t(x, y) - t_b}{t_a - t_b} &= \frac{[H - (w_y/2a)]}{[H - (w_y/a)]} \times \exp\left[\left(H^2 - \frac{Hw_y}{a}\right)\frac{xa}{\bar{w}_x} + Hy\right] \\ &\times \operatorname{erfc}\left[\left(H - \frac{w_y}{2a}\right)\sqrt{\left(\frac{xa}{\bar{w}_x}\right)}\right] + \tfrac{1}{2}\operatorname{erfc}\left[\frac{w_y}{2a}\sqrt{\left(\frac{xa}{\bar{w}_x}\right)} - \frac{y\sqrt{(\bar{w}_x)}}{2\sqrt{(ax)}}\right] - \frac{H\exp[(w_y/a)y]}{2[H - (w_y/a)]} \\ &\times \operatorname{erfc}\left[\frac{w_y}{a}\sqrt{\left(\frac{xa}{\bar{w}_x}\right)} + \frac{y}{2}\sqrt{\left(\frac{\bar{w}_x}{xa}\right)}\right].\end{aligned}\right\} \text{(III)}$$

From equation (III) the solution of equation (24) is obtained if we assume that $w_y = 0$.

From equation (III) the following formulae are obtained

$$N = \frac{Nu_x\sqrt{(\pi)}}{\sqrt{(\overline{Pe}_x)}} = \frac{\phi(K, B) - \tfrac{1}{2}\sqrt{(\pi)}\,B\operatorname{erfc}\tfrac{1}{2}B}{\{[1 - (B/K)] - (1/K\sqrt{\pi})\,\phi(K, B) + \tfrac{1}{2}(B/K)\operatorname{erfc}\tfrac{1}{2}B\}} \qquad \text{(IV)}$$

where

$$\phi(K, B) = [1 - \tfrac{1}{2}(B/K)]\sqrt{(\pi)}\,K \times \exp(K^2 - BK)\operatorname{erfc}(K - \tfrac{1}{2}B) \qquad \text{(V)}$$

$$K = \frac{Hx}{\sqrt{(\overline{Pe}_x)}}, \quad B = \frac{Pe_x^*}{\sqrt{(\overline{Pe}_x)}} = \frac{w_y\sqrt{x}}{\sqrt{(a\bar{w}_x)}}. \qquad \text{(VI)}$$

The value N_b is

$$N_b = \frac{\sqrt{(\pi)}Nu_{xb}}{\sqrt{(\overline{Pe}_x)}} = \frac{1}{[1 - (B/K)]}[\phi(K, B) - \tfrac{1}{2}\sqrt{(\pi)}\,B\operatorname{erfc}\tfrac{1}{2}B]. \qquad \text{(VII)}$$

If the effect of transverse mass transfer is neglected ($B = 0$), from formulae (IV) and (VII) we obtain formulae (29) and (33) respectively since

$$\phi(K, 0) = \sqrt{(\pi)}K\exp K^2\operatorname{erfc}K = f(K). \qquad \text{(VIII)}$$

When deepening of the evaporation surface does not occur ($\xi = 0$, $K = \infty$), we obtain from formulae (IV) and (V)

$$N = N_b = \exp\left(-\frac{B^2}{4}\right) - \tfrac{1}{2}\sqrt{(\pi)}B\operatorname{erfc}\tfrac{1}{2}B \qquad \text{(IX)}$$

i.e. we obtain formula (II) since

$$[\phi(K, B)]_{K\to\infty} = \exp\left(-\frac{B^2}{4}\right).$$

Over the range of $K(0{\cdot}5 < K < 5)$, the above values of Re and the evaporation rates j_s the value of $\tfrac{1}{2} - (B/K) \leqslant 0{\cdot}06$. Thus for approximate calculations which are sufficient for engineering practice the effect of transverse mass flow may be neglected.

Abstract—The effect of transverse mass flow on heat transfer in a moist capillary-porous plate with laminar flow is analysed. The laws of heat and mass transfer with liquid evaporation from capillary-porous bodies are established. It is shown that when an evaporating surface is submerged heat-transfer coefficients are greater than those with evaporation from body surfaces.

Decrease in the heat-transfer coefficient with evaporation intensity, reported in some works on transpiration cooling, can be explained by the methods of calculation.

Résumé—On a étudié l'effet d'une injection transversale sur les échanges thermiques d'une plaque a porosité capillaire, humide, placée dans un écoulement laminaire. Les lois du transport de masse et de chaleur avec évaporation ont été établies. Les coefficients de transmission de chaleur sont plus grands lorsque la surface d'évaporation est incluse que lorsque l'évaporation se fait à la surface du corps.

L'accroissement du coefficient d'échange thermique avec l'intensité de l'évaporation, noté dans quelques travaux sur le refroidissement par injection, peut s'expliquer par le calcul.

Zusammenfassung—Der Einfluss eines Massenquerstroms auf den Wärmeübergang an einer feuchten kapillarporösen Platte bei Laminarströmung wird analysiert. Es werden Gleichungen eingeführt für den Wärme- und Stoffübergang an porösen Körpern bei Verdampfung von Flüssigkeit. Die Wärmeübergangskoeffizienten an einer in die Flüssigkeit vollständig eingetauchten, dampfbildenden Oberfläche sind grosser als die, nichteingetauchter dampfbildender Oberflächen.

Die Vergrösserung des Wärmeübergangskoeffizienten mit der Verdampfungsintensität, von der in Arbeiten über Schwitzkühlung berichtet wird, lässt sich an Hand der Rechnung erklären.